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Isocurvature Fluctuations in Tracker Quintessence Models

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Abstract

We consider the effects of the isocurvature perturbation in the tracker-type quintessence models. During the inflation, fluctuations in the amplitude of the quintessence field are generated, which provide isocurvature component of the cosmic density perturbation. Contrary to the conventional notion, it is shown that effects of the isocurvature fluctuation may become sizable in some case, and in particular, the cosmic microwave background angular power spectrum at low multipoles may be significantly enhanced due to the effect of the isocurvature mode. Such an enhancement may be detectable in the future satellite experiments.

Recent cosmological observations suggest that there exists a dark energy which must be added to the matter density in order to reach the critical density [1]. Although the cosmological constant is usually assumed as the dark energy, in the past years, a slowly evolving scalar field, dubbed as "quintessence" has been proposed as the dark energy [2]. There are some differences between the cosmological constant and quintessence. Firstly, for the quintessence, the equation-of-state parameter $\omega_Q = \rho_Q/p_Q$ varies with time, whilst for the cosmological constant, it remains a fixed value $\omega_\Lambda = -1$. Secondly, since the quintessence is a scalar field, it fluctuates.

One of the observational discriminator of these models is the cosmic microwave background (CMB) anisotropies. Many authors have studied the effects of the quintessence field on the CMB anisotropy in various models. As mentioned above, the quintessence field fluctuates, so its fluctuations should be taken into account in calculating the CMB anisotropy. Generally, the initial fluctuations of the quintessence fields are generated during inflation. Since these fluctuations behave as the isocurvature mode, there can exist both adiabatic and isocurvature perturbations. But it has been widely believed that the primordial isocurvature fluctuation damps in a period that the quintessence field rolls down the potential [3]. If this damping is efficient, the isocurvature fluctuation dose not affect the CMB anisotropy. But recently, it was pointed out that, for the cosine-type quintessence model, the isocurvature fluctuations may be detectable in future satellite experiments by the present authors [4].

In this letter, we focus on the effects of the isocurvature fluctuations on the CMB anisotropy in the tracker-type quintessence models. The tracker-type quintessence models have attractor-like solutions which alleviate the initial condition problems. When we take initial conditions such that the quintessence field starts to roll down the potential at early epoch, damping effect is significant. So, in this case, even if we take a nonzero value of the quintessence field fluctuation initially, the fluctuation goes to zero because of the damping effects. But we will show that, in some cases, the damping effect is not so significant. In such cases, the effect of the isocurvature fluctuations may be sizable and affect the CMB angular power spectrum at low multipoles.

Since the effects of the isocurvature fluctuations depend on models of quintessence, first, we classify the tracker-type quintessence models. Although many models have been proposed [2, 3, 5–19], they can be classified into two groups, using the evolution of the energy density of the quintessence ρ_Q. The evolution of the energy density of the tracker-type models can be written in a simple form. Tracker-type quintessence models, as previously mentioned, have an attractor-like solution in a sense that a very wide range of initial conditions converge to a common evolutionary track. We call the epoch when this attractor-like solution is realized as "tracking regime." In the tracking regime, ω_Q is almost constant and $1-\omega_Q^2$ is significantly different from zero [20]. This means that the kinetic and potential energy of the quintessence have a fixed ratio. Thus, the relation $\dot{Q}^2 \propto V(Q)$ holds in the tracking regime. When this relation is satisfied, we can show that the energy density of the quintessence is proportional to a^{-n}, where n is a constant. On the contrary, the energy density of the dominant component of the universe is written as $\rho_D \propto a^{-m}$ (for the matter and radiation dominations, $m = 3$ and 4, respectively). When $n = m$, the energy density of the quintessence can be a significant fraction of the total energy density of the universe from earlier epoch. We call this type of quintessence model "type-I." When $n < m$, ρ_Q cannot be significant at early epoch. We call this type of model "type-II."#1 For a purpose of this letter, we will consider two models as representative models; "AS model [5]" and "Ratra-Peebles model [12]" for the type-I and the type-II models, respectively.

Before we show the evolution of the quintessence energy density from numerical calculations, we investigate it analytically. To study the evolution of the quintessence fields, we write down the basic equations. In a spatially flat homogeneous universe which contains a perfect fluid with energy ρ and pressure p and a quintessence field Q with energy ρ_Q and pressure p_Q, the Friedmann equation becomes

$$H^2 = \frac{1}{3M_{pl}^2}(\rho + \rho_Q), \tag{1}$$

where H is the Hubble parameter and $M_{pl} \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck scale. The energy density and pressure of the quintessence field are

$$\rho_Q = \frac{1}{2}\dot{Q}^2 + V(Q), \qquad p_Q = \frac{1}{2}\dot{Q}^2 - V(Q), \tag{2}$$

where the dot represents the derivative with respect to time t. The quintessence field Q obeys the equation of motion

$$\ddot{Q} + 3H\dot{Q} + \frac{dV}{dQ} = 0. \tag{3}$$

The energy conservation equation for each component is, in case where there is no energy exchange

$$\dot{\rho} = -3H(\rho + p). \tag{4}$$

Thus, when the universe is dominated by a perfect fluid with the energy density $\rho_D \propto a^{-m}$, the scale factor becomes as

$$a \propto t^{2/m}. \tag{5}$$

In this case, from Eqs.(2) and (3),

$$\ddot{Q} + \frac{6}{mt}\dot{Q} + \frac{dV}{dQ} = 0, \tag{6}$$

#1 The case with $n > m$ is also possible [21], but almost all model which have been proposed so far, can be categorized into the type-I or the type-II models. So we will not consider the positive α case in this letter.

First, let us discuss the evolution of ρ_Q for the AS model (representative of the type-I model) which has the potential of the form

$$V(Q) = \left[(Q-b)^2 + a\right] e^{-\lambda Q} = f(Q)e^{-\lambda Q}, \tag{7}$$

where a, b and λ are model parameters. Since $f(Q)$ changes more slowly than $e^{-\lambda Q}$ in the tracking regime, in the analytic investigations below, we approximately treat $f(Q)$ as a constant. Substituting Eq.(7) into Eq.(6), we obtain

$$\ddot{Q} + \frac{6}{mt}\dot{Q} - f(Q)\lambda e^{-\lambda Q} = 0. \tag{8}$$

The solution to this equation is $Q = A \ln \lambda B t$, where A and B are constants. In the tracking regime, $\dot{Q}^2$ is proportional to $V(Q)$, thus $\rho_Q \propto t^{-2}$ in this model. Using Eq.(5), it is shown that

$$\rho_Q \propto a^{-m}. \tag{9}$$

Therefore, in the AS model, the energy density of the quintessence has the same scale factor dependence as that of the dominant component of the universe. This means that the energy density of the quintessence can be a sizable fraction of the total energy density of the universe at early epoch. In fact, the above result does not depend on the detailed structure of the function $f(Q)$ as far as $f(Q)$ is a slowly varying function on the tracking regime. Thus, the same discussion is applicable to other types of quintessence models with $V(Q) = f(Q)e^{-\lambda Q}$.

Next, we consider the Ratra-Peebles model (representative of the type-II model) which has the potential of the form,

$$V(Q) = \frac{\Lambda^{4+\alpha}}{Q^\alpha}, \tag{10}$$

where Λ and $\alpha(> 0)$ are model parameters. With this potential, the equation of motion of the quintessence field becomes

$$\ddot{Q} + \frac{6}{mt}\dot{Q} - \alpha\frac{\Lambda^{4+\alpha}}{Q^{\alpha+1}} = 0. \tag{11}$$

This equation has the solution $Q = Ct^\nu$ where $\nu = 2/(2+\alpha)$ and C is a constant. Since α is taken to be positive, the value of ν is $0 < \nu < 1$. The energy density of the quintessence in this model becomes

$$\rho_Q \propto a^{-m(1-\nu)}. \tag{12}$$

Thus, in the Ratra-Peebles model, ρ_Q decreases more slowly than that of the dominant component of the universe, which implies that the energy density of the quintessence cannot be a significant fraction of the energy density in the early universe.

Now we show evolutions of the energy density of the quintessence models from the numerical calculations. In Fig.1, we show the evolution of the quintessence energy density for the AS model, where we take several values of the initial amplitude for the quintessence field. From the figure, we can read off when the quintessence field starts to enter the tracking regime for several values of the initial amplitudes. We can also see the property of the type-I models; the energy density of the quintessence field can have $O(10\%)$ contributions to the total energy density of the universe from early epoch.

In Fig.2, the evolution of the energy density are shown for the Ratra-Peebles model. Different from the type-I models, the quintessence energy density of this type of model is negligible for $z \gg O(1)$. This fact has an important implication to the effects of the isocurvature fluctuations on the CMB angular power spectrum.

Next let us discuss the effects of the isocurvature fluctuations on the CMB angular power spectrum. For this purpose, we study the evolutions of the fluctuations of the quintessence analytically. We decompose the quintessence field Q as

$$Q(t, \vec{x}) = \bar{Q}(t) + q(t, \vec{x}), \tag{13}$$

where q is the perturbation of the amplitude of the quintessence field. The equation of motion for q is, in the synchronous gauge,

$$\ddot{q} + 3H\dot{q} - \left(\frac{a}{a_0}\right)^{-2} \partial_i^2 q + V''(\bar{Q})q = -\frac{1}{2}\dot{h}\dot{\bar{Q}}, \tag{14}$$

where the perturbed line element in the synchronous gauge is given by

$$ds^2 = -dt^2 + \left(\frac{a}{a_0}\right)^2 (\delta_{ij} + h_{ij}) dx^i dx^j. \tag{15}$$

Here a is the scale factor at time t, a_0 the scale factor at the present time, and h is the trace of h_{ij}. In the momentum space, we expand h_{ij} as

$$h_{ij}(t, \vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} \left[\frac{k_i k_j}{k^2} h(\vec{k}, t) + \left(\frac{k_i k_j}{k^2} - \frac{1}{3}\delta_{ij}\right) 6\eta(\vec{k}, t)\right] e^{i\vec{k}\vec{x}}, \tag{16}$$

with $k^2 = k_i k^i$. Notice that, in the momentum space, the gauge-invariant variable Ψ is related to h and η as

$$\Psi(k) = \frac{1}{2k^2} \left(\frac{a}{a_0}\right)^2 \left[\ddot{h} + 6\ddot{\eta} + 2H(\dot{h} + 6\dot{\eta})\right]. \tag{17}$$

From Eq.(14), we can show the damping behavior of the quintessence field in the tracking regime. To solve this equation, we write down the second derivative of the potential as a function of the sound speed c_s^2 of a quintessence field Q [3]

$$\frac{d^2V(\bar{Q})}{dQ^2} = \frac{3}{2}H\dot{c_s^2} + \frac{3}{2}H^2(c_s^2 - 1)\left[\frac{\dot{H}}{H^2} - \frac{3}{2}(c_s^2 + 1)\right], \tag{18}$$

where the sound speed of the quintessence field is written as

$$c_s^2 = \frac{\dot{p}_Q}{\dot{\rho}_Q} = \frac{\dddot{Q} - dV/dQ}{\dddot{Q} + dV/dQ}. \tag{19}$$

Since the kinetic energy is proportional to the potential energy in the tracking regime, the relation $\ddot{Q} \propto dV/dQ$ holds. So the sound speed of the quintessence field c_s^2 is a constant during this regime. Therefore $\dot{c}_s^2$ can be set to zero in Eq.(18). When the quintessence is a subdominant component of the universe, the Hubble parameter H can be written using the equation-of-state parameter $\omega_D = p_D/\rho_D$ of the dominant component of the universe,

$$H = \frac{2}{3(1+\omega_D)t}. \tag{20}$$

To study the isocurvature mode, we can neglect the right hand side of Eq.(14). Then, using the above relations, the equation of motions can be written as

$$\ddot{q} + \frac{2}{(1+\omega_D)t}\dot{q} + \frac{1}{(1+\omega_D)^2 t^2}(1-c_s^2)(c_s^2+\omega_D+2)q = 0. \tag{21}$$

This equation has power-law solutions like $q \propto t^{\xi}$, where the power-law index ξ is

$$\xi = \frac{\omega_D - 1}{2(\omega_D+1)}\left[1 \pm \sqrt{1 - \frac{4}{(\omega_D-1)^2}(1-c_s^2)(c_s^2+\omega_D+2)}\right]. \tag{22}$$

Since c_s^2 is equal to ω_Q in the tracking regime [3] and $\omega_{D,Q}$ cannot be larger than unity, the real part of ξ is always negative. Therefore q damps with time. It follows that if we take $q \neq 0$ initially, q damps to zero in case that the quintessence field experiences long period of tracking. But, in the case that the quintessence field enters the tracking regime at later time, the fluctuations may not damp so much. In this case, the isocurvature fluctuations (i.e., $q \neq 0$) can affect CMB anisotropies.

The primordial fluctuations in the quintessence amplitude is generated in the early universe, probably during the inflation. When the mass of the quintessence field is negligible compared to Hubble parameter during inflation $H_{\rm inf}$, the primordial fluctuation is [22][#2]

$$q_{\rm initial}(k) = \frac{H_{\rm inf}}{2\pi}. \tag{23}$$

For convenience, we consider the ratio of the primordial value of q to that of the gauge-invariant variable Ψ at the radiation-dominated universe:

$$r_q \equiv \frac{q_{\rm initial}}{M_{pl}\Psi}. \tag{24}$$

[#2]If $m_q \gtrsim H_{\rm inf}$, the primordial fluctuation damps to zero rapidly and cannot affect the CMB anisotropies. So we do not consider such a case.

The value of Ψ at the radiation-dominated era is [23]

$$\Psi = \frac{4}{9}\left(\frac{H_{\rm inf}^2}{2\pi|\dot{\chi}|}\right), \tag{25}$$

where χ is the inflaton field. Using the slow-roll condition for χ, we obtain

$$r_q \simeq \frac{9}{4}\frac{M_{pl}V'_{\rm inf}}{V_{\rm inf}}, \tag{26}$$

where $V_{\rm inf}$ is the inflaton potential and $V'_{\rm inf}$ is its derivative with respect to the inflaton field. For example, for the chaotic inflation with $V_{\rm inf} \propto \chi^p$ with p being an integer, r_q is given by

$$r_q|_{\rm chaotic} = \frac{9pM_{pl}}{4\chi(k_{\rm COBE})}, \tag{27}$$

where $\chi(k_{\rm COBE})$ represents the inflaton amplitude at the time when the COBE scale crosses the horizon. Numerically, we found $r_q \simeq 0.3 - 0.6$ for $p = 2 - 10$. Since the ratio r_q generically depends on the model of the inflation, we treat r_q as a free parameter in our analysis.

Now we show the results from numerical calculations. We used the modified version of CMBFAST [24] to calculate the CMB angular power spectrum C_l which is defined as

$$\langle \Delta T(\vec{x}, \vec{\gamma})\Delta T(\vec{x}, \vec{\gamma}')\rangle = \frac{1}{4\pi}\sum_l (2l+1)C_l P_l(\vec{\gamma}\cdot\vec{\gamma}'), \tag{28}$$

where $\Delta T(\vec{x}, \vec{\gamma})$ is the temperature fluctuation of the CMB pointing to the direction $\vec{\gamma}$, and P_l is the Legendre polynomial. The average is over the position $\vec{x}$.

To discuss the effects of the isocurvature fluctuations, we compare the CMB angular power spectrum in the purely adiabatic case ($q_{\rm initial} = 0$) to one with the isocurvature fluctuations ($q_{\rm initial} \neq 0$). Since the quintessence becomes the dominant component of the universe only in the recent epoch, the effect of the isocurvature mode is significant at low multipoles, in particular $l = 2$. Thus, to discuss the effect of the isocurvature fluctuations, we define the ratio

$$R_2 \equiv \frac{C_2(r_q)}{C_2(r_q = 0)}. \tag{29}$$

In Figs.4 and 5, we plot the ratio R_2 as a function of the scale factor when the tracking regime starts. Since the tracking regime approximately starts when the quintessence field begins to roll down the potential, this scale factor, denoted as a_{tr}, is defined as the scale factor when the following equation is satisfied

$$H^2 = \frac{d^2V}{dQ^2}. \tag{30}$$

The effect of the isocurvature fluctuations are larger in the AS model than in Ratra-Peebles model. This is because, in the AS model, the quintessence can have $O(10\%)$ contribution to the total energy density of the universe. This means that the isocurvature mode can affect the CMB angular power spectrum from earlier epoch. Thus, in the AS model, the isocurvature fluctuations can largely affect the CMB angular power spectrum.

Although the value of R_2 depends on models, the dependence of R_2 on a_{tr} is qualitatively same. Namely, R_2 becomes the largest when $a_{tr} \sim a_{rec}$, where a_{rec} is the scale factor at the recombination epoch. (Hereafter, the subscript "rec" is for values at the recombination.) Using the fact that the anisotropy at low multipoles is essentially determined by the Sachs-Wolfe (SW) and Integrated Sachs-Wolfe (ISW) effects, this behavior can be understood by studying the gravitational potential Ψ, in particular, at the recombination epoch. Notice that the gravitational potential is determined by the fluctuation of the total energy density which is, for purely isocurvature case, highly correlated with the energy density fluctuation of the quintessence field $\delta\rho_Q$. Thus, for larger $\delta\rho_Q|_{rec}$, Ψ_{rec} is enhanced resulting in larger R_2.

The energy density perturbation of the quintessence is given by

$$\delta\rho_Q = \dot{\bar{Q}}\dot{q} + \frac{dV}{dQ}q. \tag{31}$$

We have numerically checked that the contribution from the kinetic term is subdominant for the discussion below. When $a_{tr}/a_{rec} \gtrsim 1$, the fluctuation of the quintessence field q is frozen at the initial value. Thus, $\delta\rho_Q|_{rec}$ is proportional to $dV/dQ|_{rec}$ which becomes smaller as the initial amplitude of Q becomes larger (corresponding to larger a_{tr}). Therefore, when $a_{tr}/a_{rec} \gtrsim 1$, $\delta\rho_Q|_{rec}$ decreases as a_{tr} increases. When $a_{tr}/a_{rec} \lesssim 1$, the quintessence field is in the tracking regime at the recombination epoch. In this case, $dV/dQ|_{rec}$ is independent of a_{tr} and $\delta\rho_Q|_{rec}$ is determined by q_{rec}. As mentioned before, q damps with time when the quintessence field is in the tracking regime. Hence, q at the recombination becomes smaller for larger value of a_{tr}. Due to these two effects, R_2 takes its largest value arises when $a_{tr} \sim a_{rec}$.

The above discussion can be numerically confirmed. In Fig.6, for purely isocurvature case, we plot the evolution of the gravitational potential Ψ for several values of the initial amplitude of the quintessence field. Here, we used the AS model as an example. Importantly, Ψ_{rec} has the largest value when $a_{tr} \sim a_{rec}$.

Another feature which we can see in both models is an oscillatory behavior. This behavior can be understood from Eq.(22). When the tracking solution is realized, we can show that the imaginary part of ξ exists in this case. So the fluctuations of the quintessence field damps with oscillations. The oscillatory behavior of R_2 arises from this.

At low multipoles, "cosmic variance" dominates the uncertainty of the CMB data. The cosmic variance gives an uncertainty as large as $\sqrt{2/(2l+1)}$ to the l-th multipole. For C_2 which is mostly enhanced by the isocurvature fluctuations, we have 60 % error to the observation.[#3] Generally, the effect of the isocurvature fluctuations is larger in the type-I

[#3] The authors of [25] argue further reductions of the cosmic variance using the CMB polarization toward models than in the type-II models. In addition, we may not detect the enhancement by the isocurvature fluctuations in the type-II model if we take $r_q \lesssim 1$. This fact can be used to discriminate the models of the quintessence.

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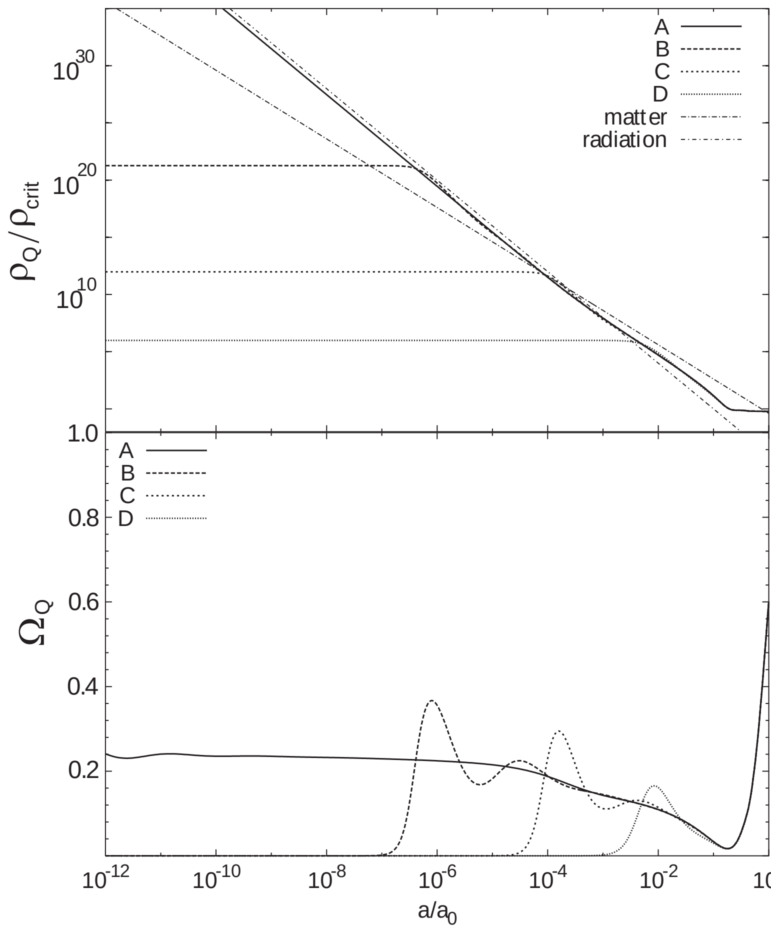

Figure 1: Evolution of the energy density in the AS model. The model parameters we take here, $\lambda = 4.0/M_{pl}$, $a = 0.01M_{pl}^2$ and $b = 68.052M_{pl}$. The initial amplitudes of the quintessence field are $40M_{pl}$ for (A), $58M_{pl}$ for (B) $63M_{pl}$ for (C) and $66M_{pl}$ for (D). The cosmological parameters are taken to be $\Omega_0 = 0.4$, $\Omega_b h^2 = 0.019$ and $h = 0.65$, where h is the Hubble parameter in units of 100 km/s/Mpc.

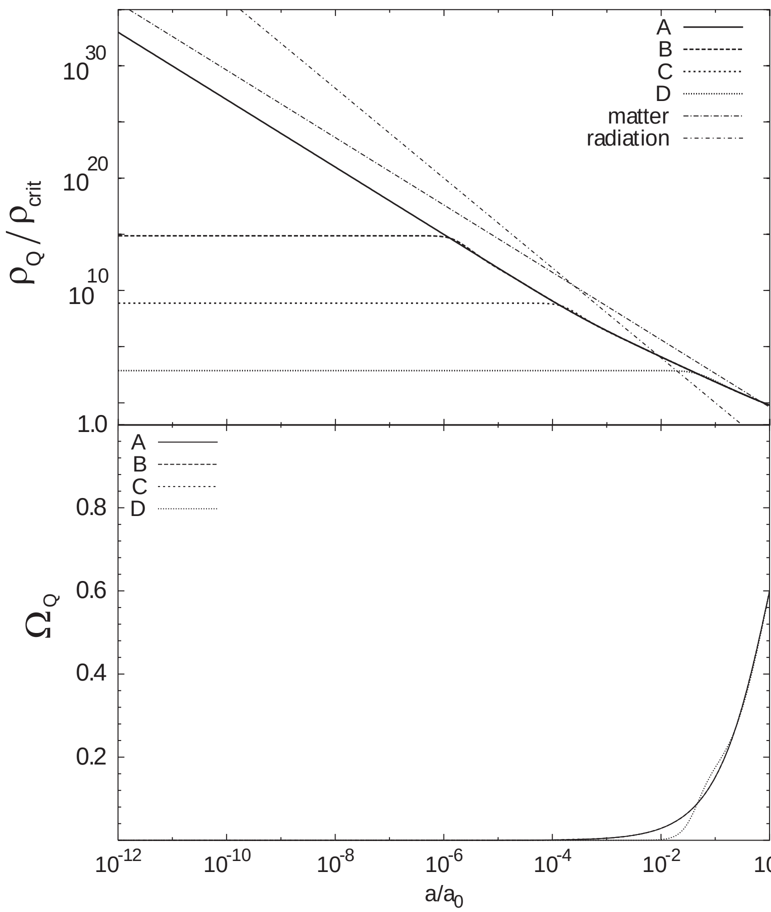

Figure 2: Evolution of the energy density in the Ratra-Peebles model. The parameter we take here, $\Lambda = 4.7 \times 10^6$ GeV and $\alpha = 6$. The initial amplitudes of the quintessence field are $1 \times 10^{-8} M_{pl}$ for (A), $1 \times 10^{-2} M_{pl}$ for (B) $0.1 M_{pl}$ for (C) and M_{pl} for (D). The cosmological parameters are the same in Fig.1.

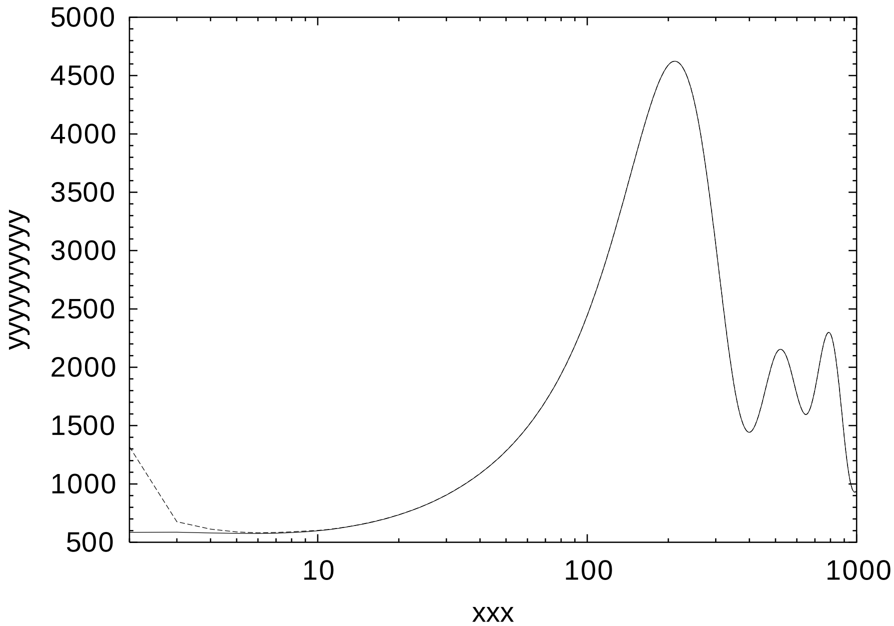

Figure 3: The CMB angular power spectrum in the AS model. The cosmological and model parameters are the same as Fig.1. Here we take $r_q = 0$ (solid line) and $r_q = 0.4$ (dashed line), the initial amplitude corresponds to $a_{tr}/a_0 \sim 4 \times 10^{-3}$.

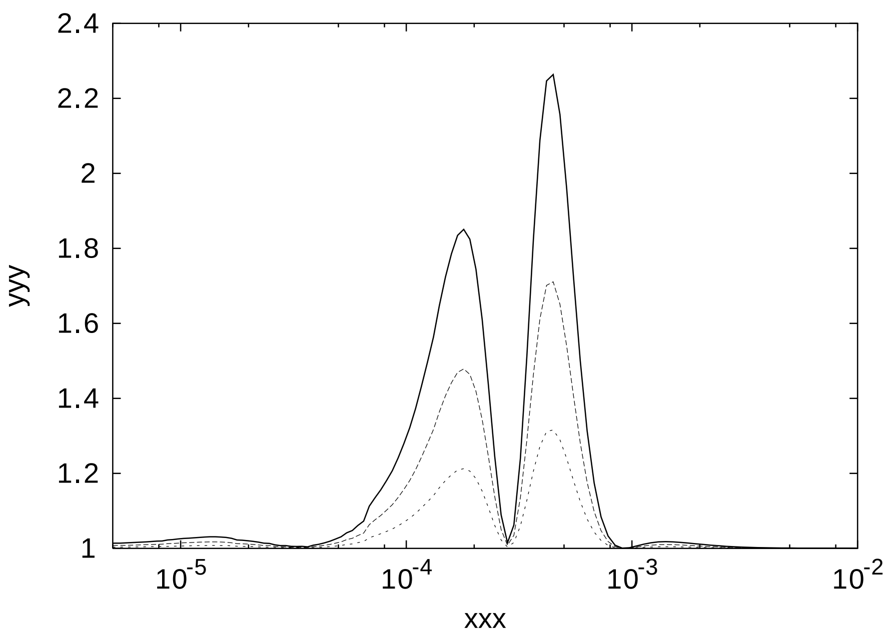

Figure 4: The ratio R_2 in the AS model. Here we take r_q to be 0.4 (solid line), 0.3 (dashed line) and 0.2 (dotted line). The cosmological and model parameters are the same as Fig.1.

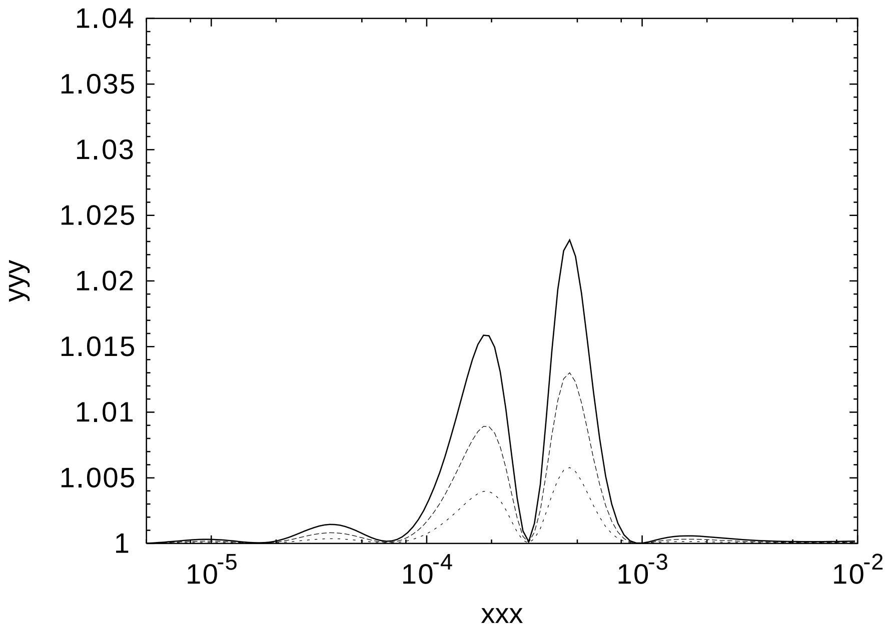

Figure 5: The ratio R_2 in the Ratra-Peebles model. We take r_q to be 0.4 (solid line), 0.3 (dashed line) and 0.2 (dotted line) Cosmological and model parameters are the same as Fig.2.

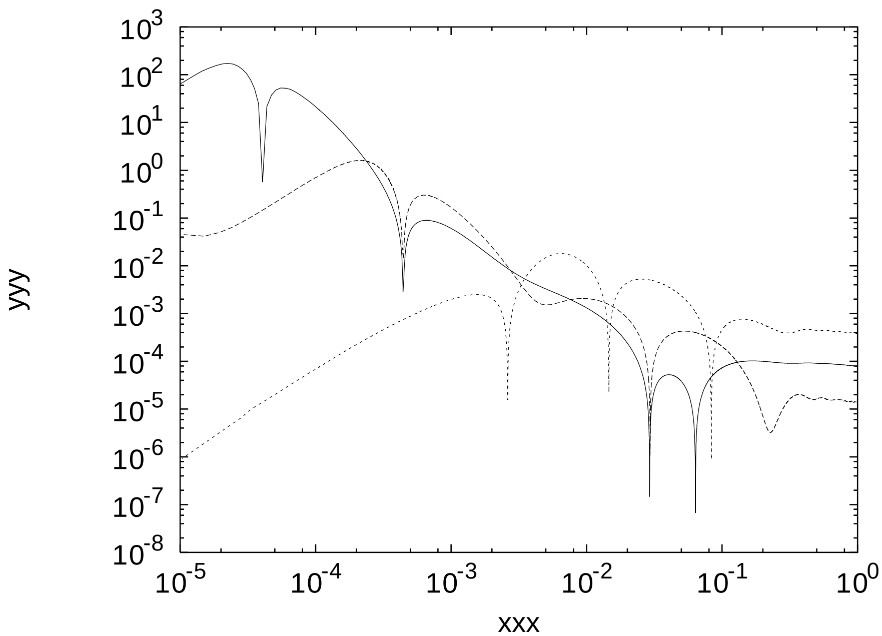

Figure 6: Evolution of the gravitational potential $\Psi(k)$ in the AS model for purely isocurvature case. Here $k = 1 \times 10^{-4} h\mathrm{Mpc}^{-1}$, and the cosmological and model parameters are the same as Fig.1. The initial amplitudes of the quintessence are $Q_{initial} = 62M_{pl}$ (solid line), $64M_{pl}$ (dashed line) and $66M_{pl}$ (dotted line).